**7.1** A horizontal-axis wind turbine with a 20-m diameter rotor is 30-% efficient in 10 m/s winds at 1-atm of pressure and 15°C temperature.

a. How much power would it produce in those winds?

b. Estimate the air density on a 2500-m mountaintop at 10° C?

c. Estimate the power the turbine would produce on that mountain with the same windspeed assuming its efficiency is not affected by air density.

a. $P_{WIND} = \frac{1}{2}\rho A v^3$

$P_{TURBINE} = \frac{1}{2}\eta \rho A v^3 = .3\left(1.225 \frac{kg}{m^3}\right)\underbrace{\left(\pi (10)^2\right)}_{\text{TURBINE BLADE SWEEP AREA}}\left(10 \frac{m}{s}\right)^3$

$= \frac{.3}{2}(1.225)(314.159)1000$

$P_{TURBINE} = \underline{\underline{57.7\ kW}}$ ANS.

$\rho_0 = 1.225 \frac{kg}{m^3}$ @ 1 ATM, 20°C

25°C = 298°K

10°C = 283°K

1 ATM = SEA LEVEL = 0 m.

b. $\rho = \rho_0 e^{\frac{-.0342 \cdot z}{T}}$

$\rho_0$ @ 1 ATM.

$\rho$ CONVERSION:

$\rho = \frac{351.1\ p(ATM)}{T(K)}$

$\Rightarrow \rho = \underbrace{\frac{353.1}{T} e^{\left(\frac{-.0342 \cdot z}{T}\right)}}_{\frac{kg}{m^3}}$

$= \frac{353.1}{283} e^{\frac{-.0342(2500)}{283}}$

$= .922 \frac{kg}{m^3}$ } AIR DENSITY @ 2500 m, 10°C

c. $P_{TURBINE} = \frac{.3}{2}(.922)(314.155)1000$

$= \underline{\underline{43.5\ kW}}$ ANS $\Leftarrow$ EXPECTED TURBINE POWER @ 2500 m, 10°C

**7.2** An anemometer mounted 10 m above a surface with crops, hedges and shrubs, shows a windspeed of 5 m/s. Assuming 15°C and 1 atm pressure, determine the following for a wind turbine with hub height 80 m and rotor diameter of 80 m:

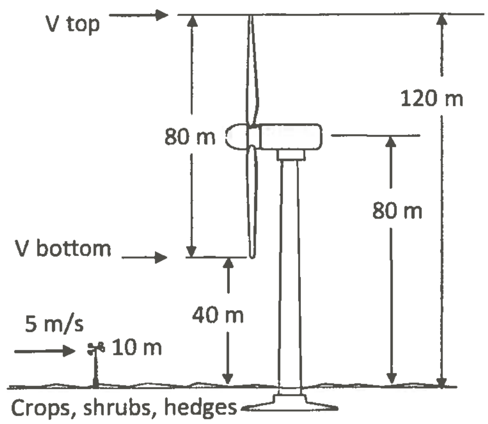


**Figure P 7.2**

**a.** Estimate the windspeed and the specific power in the wind (W/m²) at the highest point that the rotor blade reaches. Assume no air density change over these heights.

**b.** Repeat (a) at the lowest point at which the blade falls.

**c.** Compare the ratio of wind power at the two elevations using results of (a) and (b) and compare that with the ratio obtained using (7.20).

**d.** What would be the power density at the highest tip of the blade if we include the impact of elevation on air density. Assume the temperature is still 15°C. Does air density change seem worth considering in the above analysis?

$V$ @ 10 m $= 5\frac{m}{s}$

NOTE $\rho = 1.225 \frac{kg}{m^3}$ @ 15°C, 1 ATM

a) HIGH POINT OF BLADE SWEEP

$\alpha_{\text{CROPS, HEDGES, SHRUBS}} = .2$

$$\left(\frac{V}{V_o}\right) = \left(\frac{H}{H_o}\right)^{\alpha}$$

$$V = 5\frac{m}{s}\left(\frac{120m}{10m}\right)^{.2} = \underline{\underline{8.219\frac{m}{s}}} \text{ @ } 120 m$$

ANS.

SPECIFIC POWER

$$\frac{P_{120}}{A} = \frac{1}{2}(\rho_{120})(8.219)^3 = \frac{1}{2}(1.225)(8.219)^3$$

$$= \underline{\underline{340\frac{W}{m^2}}}$$ SPECIFIC POWER @ 120 m

ANS

b) LOW POINT OF BLADE SWEEP $\alpha_{\text{CROPS, HEDGES, SHRUBS}} = .2$

$$V = V_0 \left(\frac{40}{10}\right)^{.2} = 5(4)^{.2} = 6.597 \frac{m}{s}$$ WIND SPEED @ 40m

ANS

$$\frac{P_{40}}{A} = \frac{1}{2}(\rho_{40})(6.597)^3 = \frac{1}{2}(1.225)(6.597)^3$$

$$= 175 \frac{W}{m^2}$$ SPECIFIC POWER @ 40m

ANS

c) $$\frac{P_{120}}{P_{40}} = \frac{340}{175} = 1.93$$

SAME RESULT.

USING EQN 7.20, $$\frac{P}{P_0} = \left(\frac{H}{H_0}\right)^{3\alpha}$$

$$\frac{P}{P_0} = \left(\frac{120}{40}\right)^{3(.2)} = 1.933$$ ANS

d) CONSIDER AIR DENSITY CHANGE @ 120M.

USING EQN 7.17 $$\rho\left(\frac{kg}{m^3}\right) = \frac{353.1}{T} e^{-.0342\frac{z}{T}}$$

$$\rho_{120m} = \frac{353.1}{288} e^{-\frac{.0342(120)}{288}}$$

$$= 1.2081 \frac{kg}{m^3}$$

$$\frac{P_{120}}{A} \quad \frac{1}{2}(1.2081)(8.219)^3 = 335.4 \frac{W}{m^2}$$

$$\%\Delta = \frac{340 - 335.4}{340} \times 100\% = 1.35\%$$

SMALL CHANGE

$\Rightarrow$ JUSTIFIES NEGLECTING AIR DENSITY CHANGE

ANS.

1. The cost of fuel for a small power plant is currently \$10,000 per year. The owners discount rate is 12% and fuel is projected to increase at 6% per year over the 30-yr life of the plant. What is the levelized cost of fuel?

COMPUTE : $e = 6\%$ ESCALATION RATE

$n = 30$

$d = 12\%$

$$d' = \frac{(d-e)}{(1+e)} = \frac{.12 - .06}{(1+.06)} = \frac{.06}{1.06} = .0566$$

$$\text{LEVELIZED COSTS} = A_0[\underbrace{PVF(d',n)}\cdot CRF(d,n)]$$

$$PVF(d',n) = \frac{(1+d')^n - 1}{d'(1+d')^n} = \frac{(1+.0566)^{30}-1}{(.0566)(1+.0566)^{30}}$$

$$= 14.28$$

$$CRF(d,n) = \frac{d(1+d)^n}{(1+d)^n - 1} = \frac{.12(1.12)^{30}}{(1.12)^{30}-1} = 0.1241$$

$$=$$

LEVELIZED COST = $10,000(14.28)(.1241)$

$= \$17,727$ /YEAR ANS.

COMPARED TO \$10,000 ANNUAL FUE COST w/ INFLATION.

6% ANNUAL INFLATION OVER 30 YEARS ADDS SIGNIFICANT FUEL COST.

2. Better windows for a building adds $\$3/ft^2$ of window but saves $\$0.55/ft^2$ per year in reduced heating, cooling and lighting costs. With a 12% discount rate:

a. What is the net present value (NPV) of the better windows over a 30-year period with no escalation in the value of the annual savings?

b. What is the internal rate of return (IRR) with no escalation rate?

c. What is the NPV if the savings escalates at 7%/yr due to fuel savings?

d. What is the IRR with that fuel escalation rate?

a) $NPV = \Delta A \times PVF(d, n) - \Delta P$

$$PVF(d, n) = \frac{(1+d)^n - 1}{d(1+d)^n} = \frac{(1+.12)^{30} - 1}{.12(1+.12)^{30}}$$

$$= 8.055$$

$$NPV = \$.55(8.055) - \$3 = \$1.43$$

b) IRR FROM TABLE A.1 w/ $\frac{\Delta P}{\Delta A} = \frac{\$3}{.55} = 5.45$

| | Internal Rate of Return (%) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Life (yr) | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% | 12% | 14% | 16% | 18% | 20% | 22% |
| 5 | 4.58 | 4.45 | 4.33 | 4.21 | 4.10 | 3.99 | 3.89 | 3.79 | 3.60 | 3.43 | 3.27 | 3.13 | 2.99 | 2.86 |
| 6 | 5.42 | 5.24 | 5.08 | 4.92 | 4.77 | 4.62 | 4.49 | 4.36 | 4.11 | 3.89 | 3.68 | 3.50 | 3.33 | 3.17 |
| 7 | 6.23 | 6.00 | 5.79 | 5.58 | 5.39 | 5.21 | 5.03 | 4.87 | 4.56 | 4.29 | 4.04 | 3.81 | 3.60 | 3.42 |
| 8 | 7.20 | 6.73 | 6.46 | 6.21 | 5.97 | 5.75 | 5.53 | 5.33 | 4.97 | 4.64 | 4.34 | 4.08 | 3.84 | 3.62 |
| 9 | 7.79 | 7.44 | 7.11 | 6.80 | 6.52 | 6.25 | 6.00 | 5.76 | 5.33 | 4.95 | 4.61 | 4.30 | 4.03 | 3.79 |
| 10 | 8.53 | 8.11 | 7.72 | 7.36 | 7.02 | 6.71 | 6.42 | 6.14 | 5.65 | 5.22 | 4.83 | 4.49 | 4.19 | 3.92 |
| 11 | 9.25 | 8.76 | 8.31 | 7.89 | 7.50 | 7.14 | 6.81 | 6.50 | 5.94 | 5.45 | 5.03 | 4.66 | 4.33 | 4.04 |
| 12 | 9.95 | 9.39 | 8.86 | 8.38 | 7.94 | 7.54 | 7.16 | 6.81 | 6.19 | 5.66 | 5.20 | 4.79 | 4.44 | 4.13 |
| 13 | 10.63 | 9.99 | 9.39 | 8.85 | 8.36 | 7.90 | 7.49 | 7.10 | 6.42 | 5.84 | 5.34 | 4.91 | 4.53 | 4.20 |
| 14 | 11.30 | 10.56 | 9.90 | 9.29 | 8.75 | 8.24 | 7.79 | 7.37 | 6.63 | 6.00 | 5.47 | 5.01 | 4.61 | 4.26 |
| 15 | 11.94 | 11.12 | 10.38 | 9.71 | 9.11 | 8.56 | 8.06 | 7.61 | 6.81 | 6.14 | 5.58 | 5.09 | 4.68 | 4.32 |
| 20 | 14.88 | 13.59 | 12.46 | 11.47 | 10.59 | 9.82 | 9.13 | 8.51 | 7.47 | 6.62 | 5.93 | 5.35 | 4.87 | 4.46 |
| 25 | 17.41 | 15.62 | 14.09 | 12.78 | 11.65 | 10.67 | 9.82 | 9.08 | 7.84 | 6.87 | 6.10 | 5.47 | 4.95 | 4.51 |
| 30 | 19.60 | 17.29 | 15.37 | 13.76 | 12.41 | 11.26 | 10.27 | 9.43 | 8.06 | 7.00 | 6.18 | $\boxed{5.52}$ | 4.98 | 4.53 |

$\frac{\Delta P}{\Delta A} = 5.45$

IRR = 18% ANS

c) $NPV = \Delta A \times PVF(d', n) - \Delta P$ ; $d' = \frac{d-e}{1+e} = \frac{.12-.07}{1+(.07)}$

$PVF(d', n) = \frac{(1+d')^n - 1}{d'(1+d)}$

$d' = .046$

c) (CONT)

$$PVF(d', n) = \frac{(1+.046)^{30} - 1}{.046\,(1+.046)^{30}} = 16.098$$

$$NPV = (.55)\,16.098 - 3$$

$$= \underline{\underline{\$5.84}} \text{ ANS.}$$

d) $IRR_e = IRR_o(1+e) + e = .18(1+.07) + .07$

$$= \underline{\underline{26.3\%}} \text{ ANS.}$$

**1. Explain lift on a wing and compare / contrast with forces on a turbine blade.**

Bernoulli's principle to obtain lift. Air moving over the top of the airfoil has a greater distance to travel before it can rejoin the air that took the shortcut under the foil. That means the air on top moves faster causing its pressure to be lower than that under the airfoil, which creates the lifting force that holds an airplane up Fig. 7.7a or that causes a wind turbine blade to rotate Fig. 7.7b.

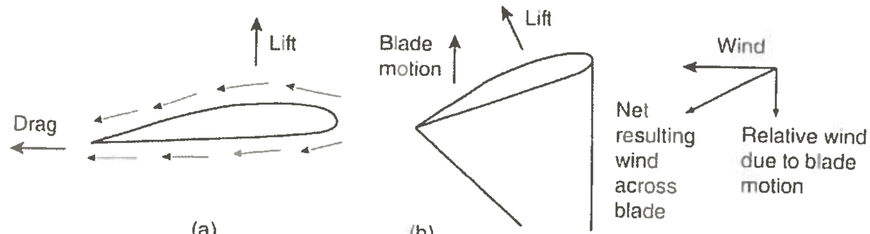


FIGURE 7.7 The lift in (a) is the result of faster air sliding over the top of the wind foil. In (b), the combination of actual wind and the relative wind due to blade motion creates a resultant that creates the blade lift.

Masters, Gilbert M. *Renewable and Efficient Electric Power Systems, 2nd Edition.* Wiley-Blackwell, 21/06/2013.

Describing the forces on a wind turbine blade is a bit more complicated than for a simple aircraft wing. A rotating turbine blade sees air moving toward it not only from the wind itself, but also from the relative motion of the blade as it rotates. As shown in Figure 7.7b, the combination of wind and blade motion is like adding two vectors, with the resultant moving across the airfoil at the correct angle to obtain lift that moves the rotor along. Since the blade is moving much faster at the tip than near the hub, the blade must be twisted along its length to keep the angles right.

**Bernoulli's principle** can be derived from the principle of conservation of energy -- in a steady flow, the sum of all forms of energy in a fluid along a streamline is the same at all points on that streamline. This requires that the sum of kinetic energy, potential energy and internal energy remains constant.

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}, \quad \textit{where}$$

$v \triangleq$ is the fluid flow speed at a point on a streamline,
$g \triangleq$ is the acceleration due to gravity,
$z \triangleq$ is the distance above a reference plane, with the + z-direction pointing opposite $g$,
$p \triangleq$ is the pressure at the chosen point,
$\rho \triangleq$ is the fluid density at all points in the fluid,

The constant on the right-hand side of the equation depends only on the streamline chosen, whereas $z$, $v$, and $\rho$ depend on the particular point on that streamline.

The following assumptions must be met for this Bernoulli equation to apply:

- the flow must be steady, i.e. the fluid velocity at a point cannot change with time,
- the flow must be incompressible – even though pressure varies, the density must remain constant along a streamline;
- friction by viscous forces must be negligible.
- Flow along a streamline, i.e. the fluid element has to be along the flow

Thus an increase in the speed of the fluid – implying an increase in both its dynamic pressure and kinetic energy – occurs with a simultaneous decrease in (the sum of) its static pressure, potential energy and internal energy.